Is unemployment hysteretic or structural? A Bayesian model selection approach for Brazil, Colombia, and Mexico*

Pedro Clavijo-Cortes†

This document estimates an unobserved components model to examine the connection between the business cycle and the natural rate of unemployment in Brazil, Colombia, and Mexico. We inquire on the possibility that the unemployment rate in these countries exhibits hysteresis and its nature. Hysteresis is defined as a dynamic structure in which the cyclical component of the unemployment rate has permanent effects on the natural component. The empirical specification is cast into a Bayesian state-space form and estimated using Markov Chain Monte Carlo methods. The specification allows for time-varying hysteresis and stochastic volatility. We use a Bayesian model selection approach to deal with the non-regular test for the null hypothesis of no time variation in the hysteresis parameter and the variances of the innovations. The results suggest an absence of hysteresis in the unemployment rate in Colombia, supply-driven hysteresis effects in Brazil, and demand-driven hysteresis effects in Mexico. Finally, the document discusses policy implications of findings regarding the degree of development in these countries.

Keywords: Hysteresis, Unobserved components, Bayesian model selection, Stochastic volatility, Time-varying parameter

JEL: C32, E24, O54

*With the usual disclaimer, I want to thank Ivan Mendieta-Muñoz for comments on a previous draft
†Ph.D. candidate in economics, University of Utah. pedro.clavijo@utah.edu

1 Introduction

The question of what lies behind the different unemployment rates in Latin America remains a subject of intense scrutiny. Why unemployment is substantially high in some economies or relatively stable and low in others are questions of tremendous policy implications. Economic theory suggests that labor market outcomes might be either structural or linked to hysteresis effects. In this latter case, hysteresis theories might enlighten us about the best strategy of macroeconomic stabilization to reduce unemployment in the long run. On the other hand, if unemployment is structural, then policies that seek to solve certain structural problems related to the relative economic backwardness of these economies may be more effective than stabilization policies in bringing down the unemployment rate.

This document attempts to distinguish between the hypotheses of hysteresis and structural unemployment using Bayesian time-series methods. This study brings the case of three Latin American countries with different degrees of development; the medium industrialized Mexico and the more commodity-exporting Colombia, as extreme cases. In the middle, we have the-medium-industrialized-commodity-exporting Brazil. These three countries exhibit drastic differences in terms of the unemployment rate level. On the one hand, we have Mexico with a stable low unemployment rate, and, on the other, Colombia with a stubbornly high unemployment rate. Brazilian unemployment, for its part, has fluctuated constantly between high and low levels.

The concept of hysteresis was first introduced in economics to explain the considerable rise in the unemployment rate in Europe during the 1980s. However, less is known about hysteresis in developing countries, in particular, Latin American economies. Unemployment has also been a pervasive macroeconomic problem in the region, and the concept of hysteresis well might be employed to explain it. Hysteresis concept, as initially introduced by Blanchard and Summers (1986), maintains that long term unemployment, typically referred to as the natural rate, might be influenced by the vicissitudes of actual unemployment. In other words, disturbances that impact unemployment cause a departure from its natural rate and affect it persistently since those disturbances also shift the natural rate.

A key aspect not well developed in the literature is the nature of hysteresis effects. These effects are typically associated with shifts in aggregate demand, which induce unemployment to react with high persistence over time. Ball (2009) presents empirical evidence on the presence of hysteresis effects associated with shifts in aggregate demand for developed countries. However, developing countries are structurally different from developed economies, and consequently, unemployment might react with high persistence to supply shocks.

In this document, we use a structural time series model to assess if unemployment rates in Brazil, Colombia, and Mexico exhibit hysteresis and what the nature of hysteresis is. In particular, we follow Jaeger and Parkinson (1994) by employing an unobserved components model that splits the actual unemployment rate into trend and cyclical components to explore the possibility that unemployment cycles cause persistent increases in the natural rate of unemployment. Relying on the literature on hysteresis (Blanchard and Summers, 1986), the hysteresis model of Jaeger and Parkinson (1994) postulates that the cycle modifies the trend permanently; that is, that the cycle drives the trend.

The decomposition of actual unemployment into natural and cyclical components proposed here implies that the natural rate stands for the rate of unemployment that would

occur in the absence of cyclical fluctuations. Thus, the natural rate component represents the sum of structural and frictional unemployment (Rissman, 1986). Consequently, if there is no evidence of hysteresis effects, long run unemployment must be driven exclusively by structural determinants and labor market frictions. To make this document comparable to recent literature that employs multivariate unobserved components models that treat trend and cycle as latent variables (e.g., Laubach (2001) and Berger and Everaert (2008)), we denote the long run or trend unemployment as the natural rate of unemployment.

An advantage of using the bivariate approach developed by Jaeger and Parkinson (1994) is that it extends the univariate approach to incorporate more information to detect the presence of hysteresis. Thus, alternative variables can be used to identify cyclical unemployment movements to capture potential sources of hysteresis effects. Different specifications of the unobserved components model would allows us to determine if aggregate demand is the primary source of hysteresis effects or if aggregate supply can play any role. A disadvantage of Jaeger and Parkinson's (1994) approach, however, is the original model's rigidity as it does not allow to capture changes in economic conditions since the key parameters are constant. In this regard, Pérez-Alonso and Sanzo (2010) using a non-linear unobserved components approach, found that allowing for changes in the hysteresis parameter is essential to capture unemployment hysteresis dynamics.

Motivated by Pérez-Alonso and Sanzo (2010), we modify the original model of Jaeger and Parkinson (1994) and estimate the hysteresis parameter as a stochastically evolving coefficient to capture potential changes in hysteresis. Additionally, we also allow for a more flexible error structure, which is especially relevant for countries facing crises and policy changes with impacts on variability in key macroeconomic variables. We based our decision of allowing for time variation in parameters and variances on a growing body of empirical evidence that emphasizes the importance of time-varying structures in explaining the macroeconomic data (Sims and Zha, 2006).

Our approach differs from the original specification developed by Jaeger and Parkinson (1994) in considering that the scar shocks leave on the natural rate can change over time. The economic intuition of this time variation in hysteresis is that in the course of development, economic activity is punctuated by small and big crises, and therefore the impacts of business cycles on unemployment can vary over time. We suggest that the magnitude of the unemployment movements is essential to the trace shocks leave on the natural rate and that this magnitude can change throughout development. Moreover, allowing for stochastic volatility in the error term is vital to avoid any bias toward the erroneous finding of time-varying coefficient. As in the literature on the origins of the Great Moderation (Galí and Gambetti, 2009), time-varying volatility and coefficient instability are rivals explaining the data.

Jaeger and Parkinson (1994) estimate the unobserved components model using the Kalman filter and maximum likelihood. Instead, we use modern Bayesian methods to estimate our model. The reason is that we extend their original specification by allowing (but not forcing) the hysteresis parameter to change over time and stochastic processes to exhibit stochastic volatility. This extension incorporates additional issues related to model specification and estimation that are non-regular from a classical point of view, making standard Kalman filter techniques not directly applicable. Hence, we employ the Bayesian stochastic model specification search outlined in Frühwirth-Schnatter and Wagner (2010) to cope with these modeling problems.

Methodologically, this study is similar to that of Berger et al. (2016), Berger and

Kempa (2019), and Grant and Chan (2017), which employ Bayesian stochastic model specification search. These studies, however, are focused on the US case, and the question of hysteresis is absent. Pérez-Alonso and Sanzo (2010) employ Jaeger and Parkinson's (1994) approach to detect unemployment hysteresis in some developed countries. Pérez-Alonso and Sanzo (2010) also propose an extension of the original empirical procedure of Jaeger and Parkinson (1994) by allowing the hysteresis parameter to switch between regimes. Their results suggest that unemployment in the US exhibits hysteresis, thus reversing the previous results of Jaeger and Parkinson (1994) and of the bulk of empirical works that have failed to find hysteresis effects in the US (see, Nelson and Plosser (1982) and León–Ledesma (2002)). Berger and Everaert (2008) also use a Bayesian framework to estimate a structural model explaining unemployment by demand and supply factors. Their work relates to ours since both attempt to determine whether demand or supply determines unemployment persistence. They show that for the US, demand forces seem to be more dominant in explaining unemployment variation.

Importantly, the empirical literature on hysteresis in Latin American countries is scarce. Some relevant examples of research into hysteresis in Latin America include the works of Ayala et al. (2012) and Ball et al. (2013). Ayala et al. (2012) use a battery of univariate time-series procedures to distinguish between the hypotheses of hysteresis and structural unemployment. The latter is defined as a mean-reverting unemployment rate that features structural breaks. Their results support the structural unemployment hypothesis in Colombia and Mexico. On the other hand, Ball et al. (2013) examine significant and persistent increases in unemployment over time in Latin America, finding that they are caused by contractions in aggregate demand. In Colombia, the contraction in aggregate demand was caused by a severe tightening of monetary policy motivated by the central bank's desire to reduce inflation. Following the intuition on hysteresis, they conclude that these findings are evidence of unemployment hysteresis.

The present work seeks to contribute to the empirical literature on hysteresis and its nature in Latin America. The main contributions of our study can be summarized as follows. The Bayesian stochastic model specification search overwhelmingly rejects the possibility of a time-varying hysteresis parameter, suggesting that the time varying coefficient is irrelevant to capture hysteresis dynamics, in the three countries considered. In this regard, the study reveals that time-varying volatility is the primary source of instability in the data. Our results evince an absence of hysteresis in Colombia, which leads us to conclude that the high Colombian unemployment is driven by structural factors. In Mexico and Brazil, we find evidence of constant hysteresis effects, but in Mexico, aggregate demand seems to be the driving force of hysteresis, while in Brazil, hysteresis effects are supply-driven.

The remainder of the document proceeds as follows. Section 2 introduces the unobserved components model in the spirit of Jaeger and Parkinson (1994) but incorporating the features of a time-varying hysteresis parameter and stochastic volatility. Section 3 presents the Bayesian estimation, the prior setup, and the corresponding Markov Chain Monte Carlo (MCMC) algorithm, all inspired by Frühwirth-Schnatter and Wagner (2010). Section 4 presents the main results, while section 5 offers an alternative specification of the model introduced in section 2 to check for alternative sources of hysteresis effects. Finally, section 6 concludes that the hysteresis nature or its absence can be attached to the degree of development in each country.

2 The unobserved components model

This section lays down the empirical unobserved components model for the unemployment rate, similar in spirit to Jaeger and Parkinson (1994). We start from the additive decomposition of the unemployment rate (u_t) into a nonstationary trend and the transitory deviation from the trend given by

$$u_t = u_t^\tau + u_t^c, \tag{2.1}$$

where u_t^τ denotes the subjacent trend of unemployment, which might be interpreted as the natural rate component, and u_t^c stands for cyclical unemployment. Since hysteresis affects the natural rate by definition, this natural component evolves as

$$u_{t+1}^\tau = u_t^\tau + \alpha_t u_t^c + \eta_t^\tau, \quad \eta_t^\tau \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_{\eta,\tau}^2), \tag{2.2}$$

where α_t represents the fraction of the cycle integrated into the natural rate referred to as hysteresis. Hence, the coefficient captures the natural rate increases after a cyclical movement in the unemployment rate. The hysteresis model was proposed for unemployment by Jaeger and Parkinson (1994) to formalize the idea that a rise in cyclical unemployment can lead to a permanent increase in the natural rate. This model presents a permanent component with richer dynamics than a pure random walk due to the correlation of the components it suggests. However, evidence in favor of $\alpha_t = 0$ implies that the natural rate evolves therefore as a random walk, which is a typical assumption in the extant literature.

The key feature of this framework that we want to concentrate on is the evolution of α_t. We consider increasing the flexibility of the original proposal of Jaeger and Parkinson (1994) by allowing the coefficient α to vary over t. We model the varying coefficient α_t as a smooth stochastic process of the form:

$$\alpha_{t+1} = \alpha_t + \eta_t^\alpha, \quad \eta_t^\alpha \overset{i.i.d.}{\sim} \mathcal{N}(0, \sigma_{\eta,\alpha}^2). \tag{2.3}$$

Relaxing the assumption of a constant parameter may allow a better estimation of hysteresis dynamics if it exists (Pérez-Alonso and Sanzo, 2010). The random walk specification of α_t allows for a very flexible evolution of the parameter over time, particularly convenient to capture smooth transition and structural change. In the context of this study, the time-varying coefficient is convenient to connect business cycles, expressed in the form of cyclical unemployment and non-linearities.

For its part, the cyclical component of the unemployment rate, u_t^c, is specified as a zero-mean stationary autoregressive process of order two. Thus,

$$u_{t+1}^c = \beta_1 u_t^c + \beta_2 u_{t-1}^c + \exp\{h_t^c\}\eta_t^c, \quad \eta_t^c \overset{i.i.d.}{\sim} \mathcal{N}(0, 1). \tag{2.4}$$

To identify the model, the system is completed by augmenting it with an equation that connects output growth (g_t) with the unemployment cyclical component given by

$$g_t = \gamma_1 g_{t-1} + \gamma_2 u_t^c + \exp\{h_t^g\}\eta_t^g, \quad \eta_t^g \overset{i.i.d.}{\sim} \mathcal{N}(0, 1). \tag{2.5}$$

Equation (2.5) can be interpreted as a version of Okun's law, in which output growth relates to the unemployment cyclical component. In this specification, γ_2 stands for the Okun coefficient, which is expected to be negative. Thus, when the unemployment rate is above its natural rate, output growth is slowing on average. This inverse relationship between cyclical unemployment and output growth is a feature of the demand side of the economy, in which output grows during a cyclical recovery lead firms to create jobs and hire unemployed workers. Conversely, output falls in a recession trigger employment destruction, and some workers end up joining the ranks of the unemployed.

Hence, the choice of Okun's law is an effort to test for potential hysteresis effects in a framework in which unemployment interacts with aggregate demand measured by g_t. For example, considering that the economies contemplated in this study are open economies, we might expect that, in line with Thirlwall's (1979) balance-of-payments-constrained growth model, the growth in world demand accelerates income growth in these economies, causing a reduction in the unemployment rate.

Note that the choice of the additional variable needed to identify the model is relatively arbitrary. There exist alternative closure equations, but in this particular scenario, we want to examine whether hysteresis rises in the presence of changes in the aggregate demand denoted by g_t. The selection of the variable is motivated by the previous results of Ball et al. (2013) that suggest that hysteresis in Latin America is demand-driven.

Finally, β_1, β_2, and γ_1 are parameters to estimate and the stochastic volatility terms $\exp\{h_t^i\}$ are included to capture the time variation in the variances of the innovations. Stochastic volatilities are modeled as random walks

$$h_{t+1}^i = h_t^i + \eta_t^{h^i}, \quad \eta_t^{h^i} \overset{i.i.d.}{\sim} \mathcal{N}\left(0, \sigma_{\eta,h^i}^2\right), \tag{2.6}$$

for $i = c,\ g$. The random walk assumption for the stochastic volatility component is common in macroeconometrics (see, e.g., Cogley and Sargent (2005), Primiceri (2005), Cogley et al. (2010)). This assumption is in line with the smooth transition specifications used in previous research to capture highly persistent or permanent changes in volatility, such as the upsurge in volatility during crisis episodes.

3 Bayesian estimation

3.1 Stochastic model specification search

The empirical model outlined in Section 2 nests alternative models of interest. However, choosing between these alternative models poses some issues related to model specification and estimation. In particular, issues related to whether the hysteresis parameter is time-varying or constant or whether all the variances of innovations exhibit stochastic volatility. Therefore, we must determine what components to include and decide whether they are fixed or time-varying. However, model specification for state-space models is a challenge as this leads to non-regular testing problems. Consider, for instance, the question of whether α_t should be modeled as constant or varying over time. Although α_t can be filtered using the Kalman filter and the variance of the innovations $\sigma_{\eta,\alpha}^2$ can be estimated using Maximum Likelihood, the question of whether the time variation is relevant implies testing $\sigma_{\eta,\alpha}^2 = 0$ against $\sigma_{\eta,\alpha}^2 > 0$, which is a non-regular testing problem as the parameter lies on the boundary of the parameter space under the null. A similar

problem arises when testing whether the stochastic volatilities are relevant.

The Bayesian approach can be invoked to cope with such non-regular testing problems. In particular, we follow Frühwirth-Schnatter and Wagner (2010) by working with their Bayesian stochastic model specification search. This approach involves two transformations to the original model: (i) non-centering and (ii) parameter expansion. In the first transformation, the initial value is subtracted from all states such that the transformation moves the initial state and the standard deviation into the mean equation leaving no unknown parameters in the state equation. The second transformation in the approach of Frühwirth-Schnatter and Wagner (2010) entails introducing an indicator that randomly takes the values -1 or 1 to decide if a particular component of the model is fixed or changes over time. The exact implementation applied to our state-space model is outlined below.

3.1.1 Non-centered parameterization

Motivated by Frühwirth-Schnatter and Wagner (2010), we rearrange the data-generating process for the time-varying parameter α_t in equation (2.4) to remove the parameter to be restricted ($\sigma_{\eta,\alpha}$). The transformed equation is

$$\alpha_{t+1} = \alpha_0 + \sigma_{\eta,\alpha}\widetilde{\alpha}_{t+1}, \tag{3.1}$$

with $\widetilde{\alpha}_{t+1} = \widetilde{\alpha}_t + \widetilde{\epsilon}_t$, $\widetilde{\alpha}_1 = 0$, $\widetilde{\epsilon}_t \overset{i.i.d.}{\sim} \mathcal{N}(0,1)$, and α_0 as the initial value of α_{t+1}.

The resulting non-centered parameterization has the convenient property that state equation error standard deviation, $\sigma_{\eta,\alpha}$, can conditionally be treated as a standard regression coefficient with support $\mathbb{R}$ (Chan and Strachan, 2021).

A crucial aspect of the non-centered parameterization is that it introduces an identifiability problem: multiplying the components $\sigma_{\eta,\alpha}$ and $\widetilde{\alpha}_t$ by -1 will not change their product in equation (3.1). Consequently, we obtain an observationally equivalent representation characterized by the same likelihood by switching the sign to both elements. Hence, the likelihood function is symmetric around zero along the $\sigma_{\eta,\alpha}$ dimension and therefore multimodal. If the hysteresis parameter varies over time, i.e., $\sigma^2_{\eta,\alpha} > 0$, then the likelihood function will concentrate around the two modes $-\sigma_{\eta,\alpha}$ and $\sigma_{\eta,\alpha}$. However, Frühwirth-Schnatter and Wagner (2010) pointed out that this identifiability problem has no bearing on the centered model's inferences. Besides, when $\sigma^2_{\eta,\alpha} = 0$ the likelihood function will become unimodal around zero, which means that allowing for non-identification of $\sigma_{\eta,\alpha}$ provides useful information on whether $\sigma^2_{\eta,\alpha} > 0$.

Likewise, the stochastic volatility terms can be reparameterized as follows:

$$h^i_{t+1} = h^i_0 + \sigma_{\eta,h^i}\widetilde{h}^i_{t+1}, \tag{3.2}$$

where $\widetilde{h}^i_{t+1} = \widetilde{h}^i_t + \widetilde{\upsilon}^i_t$, $\widetilde{h}^i_1 = 0$, $\widetilde{\upsilon}^i_t \overset{i.i.d.}{\sim} \mathcal{N}(0,1)$.

3.1.2 The Parsimonious Specification

The non-centered parameterization is helpful for model selection both for the components and the dynamics of the model coefficients and the innovation variances. The question of whether the hysteresis parameter or the variances of innovations vary over

time or not can be expressed as a variable selection problem in equations (3.1) and (3.2). To deal with this variable selection problem, Frühwirth-Schnatter and Wagner (2010) introduce the parsimonious specification

$$\alpha_t = \alpha_0 + \lambda \sigma_{\eta,\alpha} \widetilde{\alpha}_t, \tag{3.3}$$

where λ is a binary indicator which is either 0 or 1. This binary parameter plays a variable selection role in which $\lambda = 1$ corresponds to the full model and therefore $\widetilde{\alpha}_t$ is included and $\sigma_{\eta,\alpha}$ is estimated from the data. Conversely, $\lambda = 0$ turns off the component $\widetilde{\alpha}_t$ in the model such that α_0 represents the constant parameter.

Likewise, the parsimonious non-centered parameterization of the stochastic volatility terms in equation (3.2) is given by

$$h_t^i = h_0^i + \theta^i \sigma_{\eta,h^i} \widetilde{h}_t^i, \tag{3.4}$$

for $i = c,\ g$ and where $\theta^i \in \{0, 1\}$ is another binary indicator. If $\theta^i = 0$, the component $\widetilde{h}_t^i$ drops from the model pushing the model towards a homoscedastic specification such that $(\exp\{h_0^i\})^2$ represents the constant variance. If $\theta^i = 1$ then $\widetilde{h}_t^i$ is included in the model and σ_{η,h^i} is estimated from the data. In this case $(\exp\{h_0^i\})^2$ is the initial value of the time-varying variance of η_t^i.

3.2 Prior distributions

The Bayesian estimation approach requires defining prior distributions for the model parameters of interest. In this regard, variable selection is sensitive to the prior choice. For instance, the shape and scale hyperparameters of the more commonly used inverted gamma distribution on the variances strongly influence the posterior distribution if the variance's actual value is close to zero. Since the inverted gamma is bounded away from zero, it artificially pulls probability mass away from zero. This is particularly important when interest centers on determining the variance of the innovations to a particular parameter for which we want to determine its time variability.

Following Frühwirth-Schnatter and Wagner (2010), we replace the typical inverted gamma prior on a variance parameter σ^2 with a normal prior centered at zero on σ. This avoids prior information for the parameter to drive the variables that are selected. Frühwirth-Schnatter and Wagner (2010) show that, compared to using an inverted gamma prior for σ^2, the posterior density of σ is much less sensitive to the hyperparameters of the normal distribution and is not pushed away from zero when $\sigma^2 = 0$. This makes the normal prior more suitable under model specification uncertainty than the inverted gamma prior. Berger et al. (2016) argue that another reason to select a normal prior is that centering the prior distribution at zero is appropriate for variable selection as σ is symmetric around zero.

Hence, we choose a normal prior distribution centered at zero with unit variance for $\sigma_{\eta,\alpha}$, σ_{η,h^c} and σ_{η,h^g}, and a gamma prior for $\sigma_{\eta,\tau}$. We use a gamma prior for $\sigma_{\eta,\tau}$ since compared to the conventional inverted gamma prior, the gamma prior has more mass concentrated around small values of $\sigma_{\eta,\tau}$, thus providing additional shrinkage (Grant and Chan (2017)). For each of the model parameters $\beta_1, \beta_2, \gamma_1, \gamma_2$, we set a non-informative Normal prior distribution. For the binary indicators θ^g, θ^c, and λ, we choose a uniform

TABLE I: Prior distributions of model parameters

		Prior		Percentiles	
Coefficients:		Belief	Strength	2.5%	97.5%
1st AR lag of unemployment gap	β_1	1.25	0.50	0.27	2.23
Sum of AR lags of unemployment gap	$\beta_1 + \beta_2$	0.90	0.01	0.87	0.93
AR(1) lag of output growth	γ_1	0.70	0.05	0.60	0.79
Okun coefficient	γ_2	-1.50	0.125	-1.74	-1.25
Constant hysteresis coefficient	α_0	0.50	0.25	0.01	0.99
Constant volatility of cyclical unemp.	h_0^c	log(0.60)	0.10	log(0.49)	log(0.73)
Constant volatility of output growth	h_0^g	log(0.70)	0.10	log(0.57)	log(0.85)
std. of time-varying hysteresis coef.	$\sigma_{\eta,\alpha}$	0.00	1.00	-1.96	1.96
std. of SV: cyclical unemployment	σ_{η,h^c}	0.00	1.00	-1.96	1.96
std. of SV: output growth	σ_{η,h^g}	0.00	1.00	-1.96	1.96
std. of the natural rate	$\sigma_{\eta,\tau}$	0.10	0.10	-0.19	0.19

prior distribution such that each model component has a prior probability $p_0 = 0.5$ of being included in the model. In other words, we assign a 0.5 prior probability to each of the binary indicators being one. Table I summarizes our priors selection.

The prior for the sum of the AR lags of unemployment gap is set based on the belief that the unemployment gap is persistent and stationary, which is a standard adoption in the literature (see, e.g., Grant (2018)). The small prior standard deviation of 0.01 is in compliance with that belief. The autoregressive coefficient of the output growth equation is also set to ensure stationarity, although the prior standard deviation is much less informative. Following the literature on Okun's law, the Okun coefficient is set similarly to that in the literature. To explicitly consider our uncertainty on unemployment hysteresis, a relatively uninformative prior is used for the standard deviation of the hysteresis coefficient that allows the parameter to range between roughly 0 and 1. The rest of the priors have been set similar to those in the literature (see, e.g., Berger et al. (2016)).

3.3 MCMC algorithm

We employ the Gibbs sampler to estimate our empirical model, which is an MCMC algorithm to simulate draws from the joint posterior distribution. However, the joint posterior distribution of all model's unknown parameters and unobserved states is analytically intractable. The MCMC algorithm we use produces tractable conditional distributions by reducing the complex non-linear model into a sequence of blocks for subsets of parameters/states that are tractable conditional on the other blocks in the sequence. The algorithm consists of using the Gibbs sampler in conjunction with the sampling schemes developed by Frühwirth-Schnatter and Wagner (2010), Kim et al. (1998), and Carter and Kohn (1994), and De Jong and Shephard (1995) to estimates the model on an equation-by-equation basis and thus obtain realizations from the posterior distribution of interest.

For notational convenience, denote the state vector $\psi_t = (u_t^\tau, u_t^c)$, the stochastic volatilities vector $h_t = (h_t^c, h_t^g)$, the unknown parameter vectors $\phi = (\beta_1, \beta_2, \gamma_1, \gamma_2, \sigma)$ with $\sigma = (\sigma_{\eta,\alpha}, \sigma_{\eta,h^\tau}, \sigma_{\eta,h^c}, \sigma_{\eta,h^g})$, the time-varying parameter α_t, and the model indicators $\mathcal{M} = (\lambda, \theta^c, \theta^g)$, such that each vector $\mathcal{M}$ corresponds to a model in which some coefficients are time-varying while others are not. For example, $\mathcal{M} = (1, 1, 0)$ is a model with a time-varying hysteresis coefficient, and stochastic volatility in the innovations to the

unemployment gap. Further let $x_t = (u_t, g_t)$ be the data vector. Stacking observations over time, we denote $x = \{x_t\}_{t=1}^T$ and define ψ, h, and α similarly. The posterior distribution of interest is therefore $\mathcal{P}(\psi, h, \alpha, \phi, \mathcal{M} \mid x)$. Thus, our MCMC algorithm consists of cycling through the following steps as suggested by Berger et al. (2016):[1]

1. Sample the binary indicators in $\mathcal{M}$ from $\mathcal{P}(\mathcal{M} \mid \psi, h, \alpha, x)$ marginalizing over the parameters ϕ and sample the unrestricted parameters in ϕ from $\mathcal{P}(\phi \mid \psi, h, \alpha, \phi, \mathcal{M}, x)$ while setting the restricted parameters, i.e., the elements in σ for which the corresponding component is not included in the model $\mathcal{M}$, equal to 0.

2. Sample the trend and cycle components ψ from $\mathcal{P}(\psi \mid h, \alpha, \phi, \mathcal{M}, x)$, the time-varying parameter α from $\mathcal{P}(\alpha \mid \psi, h, , \phi, \mathcal{M}, x)$, and the stochastic volatilities h from $\mathcal{P}(h \mid \psi, \alpha, \phi, \mathcal{M}, x)$.

3. Randomly permute the signs of $\sigma_{\eta,\alpha}$ and $\widetilde{\alpha}_t$, and of σ_{η,h^i} and $\widetilde{h}_t^i$ for $i = c,\ g$.

4 Estimation results

4.1 Data

We estimate the empirical model for Brazil (1992Q1-2015Q3), Colombia (1984Q1-2019Q4), and Mexico (1987Q1-2019Q4), employing seasonally adjusted quarterly data for unemployment and mean-adjusted output growth rate ($100 \times \ln(GDP_t/GDP_{t-4})$) for the most prolonged sample period according to data availability. Mexico's data comes from the OECD database, while Colombia's data is retrieved from the Central Bank of Colombia. Colombia's unemployment series before 2001Q1 corresponds to that of the seven biggest cities and from 2001Q1 onward to national unemployment. Colombian real GDP was compiled from series with different base years, using the methodology of Correa et al. (2003). As for Brazil, the unemployment rate was taken from OECD database and real GDP from CEPALSTAT. The availability of quarterly GDP determines Brazil's sample length. The MCMC algorithm was run for 40,000 iterations after a burn-in of 10,000 draws.

4.2 Results stochastic model specification search

The first approach to detect time variation is to estimate the unrestricted model with all binary indicators set to one. Thus, we draw posterior distributions for the standard deviations of the innovations to the three non-centered components of interest ($\sigma_{\eta,\alpha}, \sigma_{\eta,h^c}, \sigma_{\eta,h^g}$). If these distributions are bimodal, with low or no probability mass at zero, this can be taken as the first indication of time variation in the considered component. Figures I-III summarize the results of the variable selection procedure.

1. see Appendix I for further details

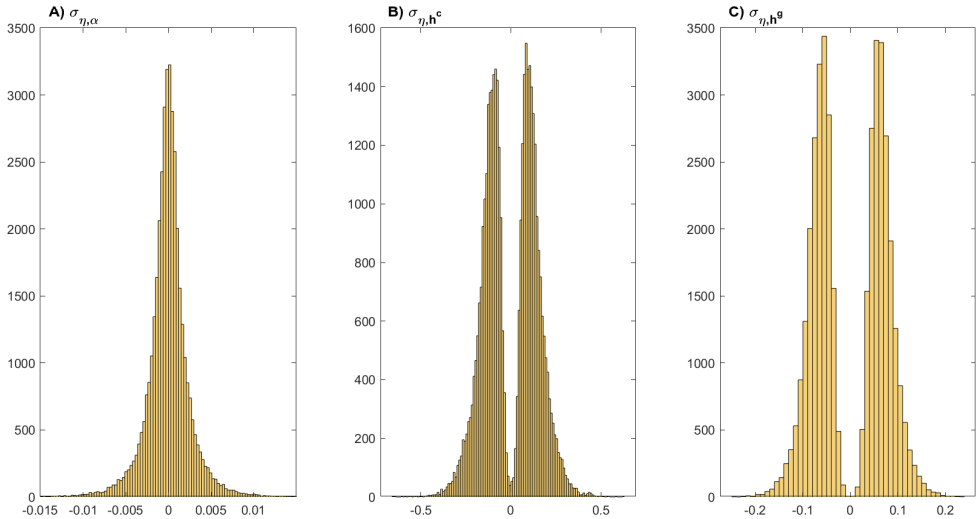

Figure I: Histograms of the posterior draws for the standard deviations of the innovations to the hysteresis parameter and stochastic volatilities for Brazil

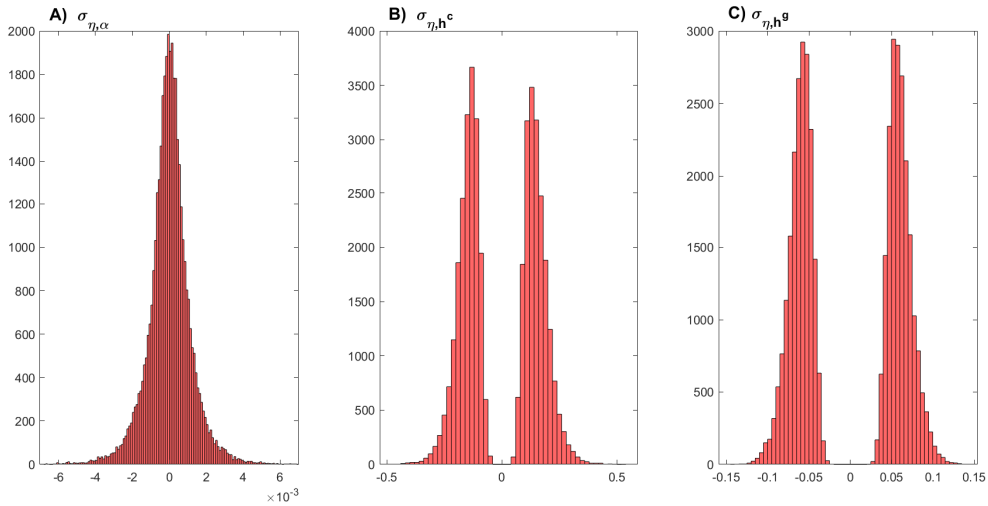

Figure II: Histograms of the posterior draws for the standard deviations of the innovations to the hysteresis parameter and stochastic volatilities for Colombia

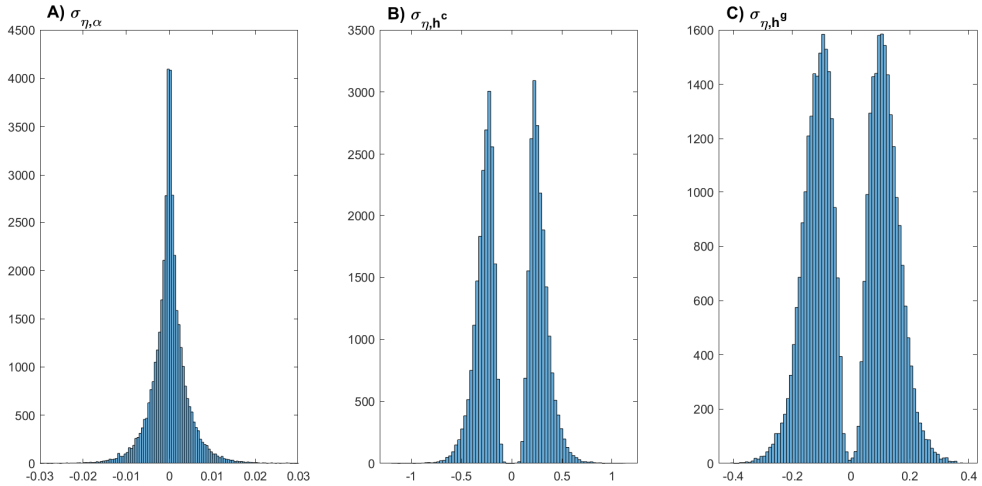

FIGURE III: HISTOGRAMS OF THE POSTERIOR DRAWS FOR THE STANDARD DEVIATIONS OF THE INNOVATIONS TO THE HYSTERESIS PARAMETER AND STOCHASTIC VOLATILITIES FOR MEXICO

The results for the three countries overwhelmingly suggest no time variation of the hysteresis parameter since the posterior distribution for $\sigma_{\eta,\alpha}$ is concentrated at zero. Hence, the evidence favors a constant hysteresis coefficient. On the other hand, we observe clear bimodality in the posterior distribution of σ_{η,h^c} and σ_{η,h^g}, suggesting time-varying variances of the innovations to the cyclical unemployment and output growth. Nonetheless, the posterior distribution of σ_{η,h^c} for Brazil presents some probability mass at zero with a bimodal pattern.

A more robust test for time variation consists of sampling the stochastic binary indicators along with the rest of the model parameters. Table II reports the individual posterior probabilities for the three binary indicators being one. These probabilities are calculated as the average selection frequencies over all iterations of the Gibbs sampler. These indicators' posterior means can be interpreted as inclusion probabilities of a particular feature due to their binary nature. Recall we assign a prior probability $p_0 = 0.5$ to each of the binary indicators being one, and normal prior distribution centered at zero with unit variance ($A_0 = 1$) for each σ.

Scott and Berger (2010) noted that when the binary indicators have a prior probability of 0.5, the fraction of selected variables will very likely be around 0.5. Thus, to provide multiplicity control for the Bayesian variable selection, we also report posterior inclusion probabilities when p_0 is set to 0.25 and 0.75. Additionally, to check robustness, we also report results over alternative values for A_0. Table III shows results for the cases in which $A_0 = 0.1$, $A_0 = 10$, and $A_0 = 100$ while holding constant the benchmark prior probability of inclusion at 0.5. The extreme case of $A_0 = 0.1$ corresponds to a relatively stronger prior that allows for less time variation, and alternatively, the case $A_0 = 100$ corresponds to prior distributions that allow for large variances on the time-varying components.

Model selection is exceptionally robust to the different specifications of the prior probability and variance in the three countries. The hysteresis parameter exhibits no time variation while variances of the shocks are time-varying. In the case of Brazil, the results are sensitive to the variance selection, particularly for the feature of time variability of the innovations to cyclical unemployment. Nonetheless, the probability of inclusion is relatively high for all choices, with a lower estimate of 0.72. In general, we conclude that

TABLE II: Posterior estimates of the indicator variables over different probabilities p_0

	Time-Varying Parameter	Stochastic Volatilities	
$A_0 = 1$	λ	θ^c	θ^g
$p_0 = 0.5$			
Brazil	0.002	0.94	1.00
Colombia	0.0008	1.00	1.00
Mexico	0.0005	1.00	1.00
$p_0 = 0.25$			
Brazil	0.0005	0.87	1.00
Colombia	0.0003	1.00	1.00
Mexico	0.0004	1.00	1.00
$p_0 = 0.75$			
Brazil	0.005	0.97	1.00
Colombia	0.003	1.00	1.00
Mexico	0.004	1.00	1.00

time variation in the variances of the shocks to the cyclical unemployment, and the output growth, are features with a high probability of being included in the model for these countries.

4.3 Parameter estimates and unobserved components

The stochastic model specification search exercise has found that the parsimonious model for Brazil, Colombia, and Mexico should include only time variation in the variances of the shocks to the cyclical unemployment and the output growth. The next step is to estimate the parameters and states employing this favored model. The estimates are shown below.

4.3.1 Posterior distribution of parameters

This subsection presents and discusses the model's parameter estimates favored by the stochastic model specification search. Table IV reports the posterior means and 2.5 and 97.5 percentiles for model parameters.

As expected, the sum of the AR lags of the unemployment gap suggests that cyclical unemployment is quite persistent as the sum equals 0.92, 0.93, and 0.90 in Brazil, Colombia, and Mexico, respectively. The AR lag of output growth implies the series behaves as a stationarity process when the unemployment gap equals zero. Regarding the Okun coefficient, the coefficient estimate in Brazil is smaller among these countries, implying that a given unemployment gap is associated with a much lower output growth rate in Brazil than in Colombia and Mexico. The estimate of -0.17 has the correct sign, although the 95% credible interval ranges from -0.47 to 0.02, including the zero value.

Interestingly, the only evidence of hysteresis effects was found in Mexico. For this country, a decrease in the cyclical unemployment rate of 1 percent leads to a permanent

TABLE III: Posterior estimates of the indicator variables over different variances A_0

	Time-Varying Parameter	**Stochastic Volatilities**	
$p_0 = 0.5$	λ	θ^c	θ^g
$A_0 = 0.1$			
Brazil	0.01	0.98	1.00
Colombia	0.01	1.00	1.00
Mexico	0.04	1.00	1.00
$A_0 = 10$			
Brazil	0.0001	0.80	1.00
Colombia	0.0001	1.00	1.00
Mexico	0.0005	1.00	1.00
$A_0 = 100$			
Brazil	0.00	0.72	1.00
Colombia	0.00	1.00	1.00
Mexico	0.00003	1.00	1.00

TABLE IV: Posterior distributions of model parameters

		Brazil			**Colombia**			**Mexico**		
		Mean	Percentiles		Mean	Percentiles		Mean	Percentiles	
Parameters:			2.5%	97.5%		2.5%	97.5%		2.5%	97.5%
1st AR lag of unemployment gap	β_1	1.25	1.18	1.33	1.16	1.09	1.24	1.19	1.10	1.27
2nd AR lag of unemployment gap	β_2	-0.33	-0.41	-0.25	-0.23	-0.31	-0.16	-0.29	-0.37	-0.21
AR(1) lag of output growth	γ_1	0.69	0.62	0.76	0.67	0.59	0.75	0.64	0.58	0.71
Okun coefficient	γ_2	-0.17	-0.47	0.02	-0.29	-0.51	-0.13	-1.77	-2.11	-1.32
Constant hysteresis coefficient	α_0	0.02	-0.03	0.11	0.01	-0.03	0.06	0.23	0.002	0.49
Constant volatility of cyclical unemployment	$\exp\{h_0^c\}$	0.56	0.47	0.66	0.55	0.46	0.64	0.53	0.45	0.63
Constant volatility of output growth	$\exp\{h_0^g\}$	0.77	0.61	0.95	0.68	0.56	0.80	0.77	0.65	0.90
std. of time-varying hysteresis coefficient	σ_{η,h^α}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
std. of SV: cyclical unemployment	σ_{η,h^c}	0.0006	-0.24	0.24	0.001	-0.19	0.19	0.0005	-0.25	0.25
std. of SV: output growth	σ_{η,h^g}	0.0005	-0.08	0.08	0.0004	-0.07	0.07	0.0003	-0.13	0.13
std. of the natural rate	σ_{η,h^τ}	0.31	0.18	0.41	0.20	0.09	0.32	0.55	0.43	0.62

decrease of the actual unemployment rate of about 0.23 percent. For the cases of Brazil and Colombia, the parameter is small, and the 95% credible interval contains the value of zero. This implies that cyclical unemployment does not affect the natural rate, and therefore its movements are determined exclusively by frictional and structural unemployment in these two countries.

4.3.2 Posterior distribution of states

In this subsection, we report the estimates of the natural rate of unemployment and the stochastic volatilities from the selected model. Figures IV-VI show the estimate of the state variables for Brazil, Colombia, and Mexico, respectively.

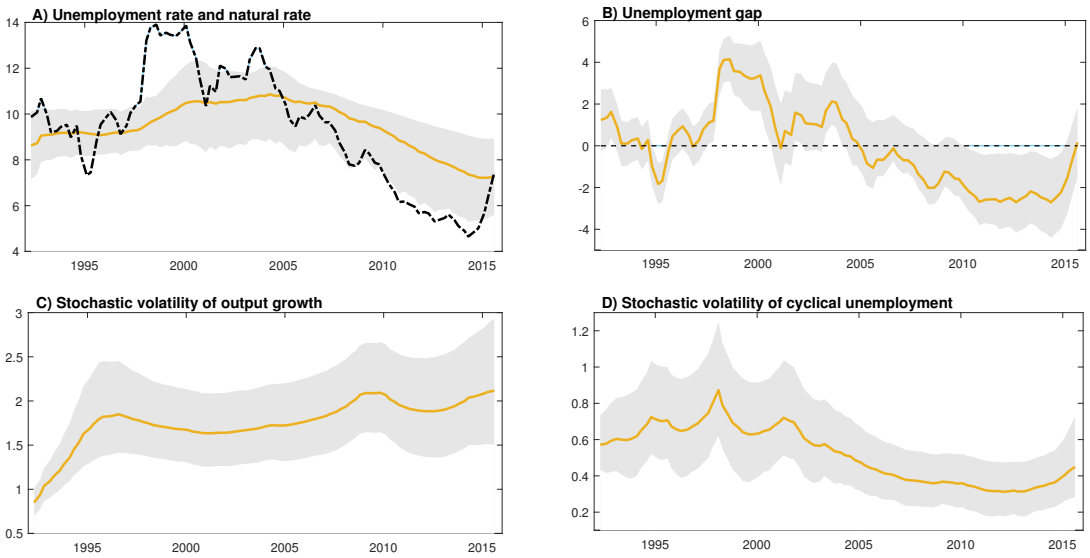

Figure IV: Estimates of the natural rate, unemployment gap, and stochastic volatilities for Brazil. The shaded region represents the 10% and 90% quantiles.

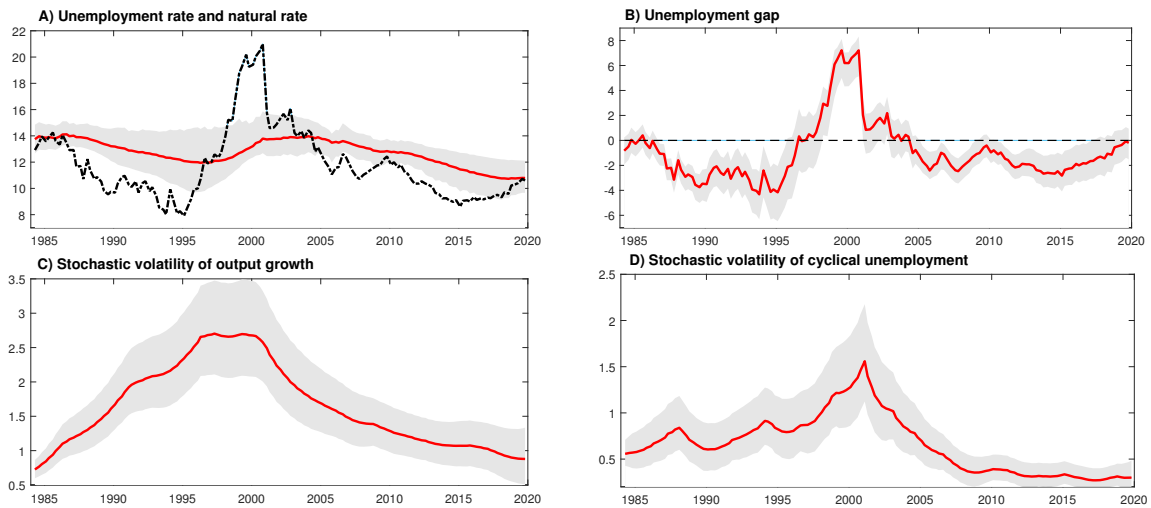

FIGURE V: ESTIMATES OF THE NATURAL RATE, UNEMPLOYMENT GAP, AND STOCHASTIC VOLATILITIES FOR COLOMBIA. THE SHADED REGION REPRESENTS THE 10% AND 90% QUANTILES.

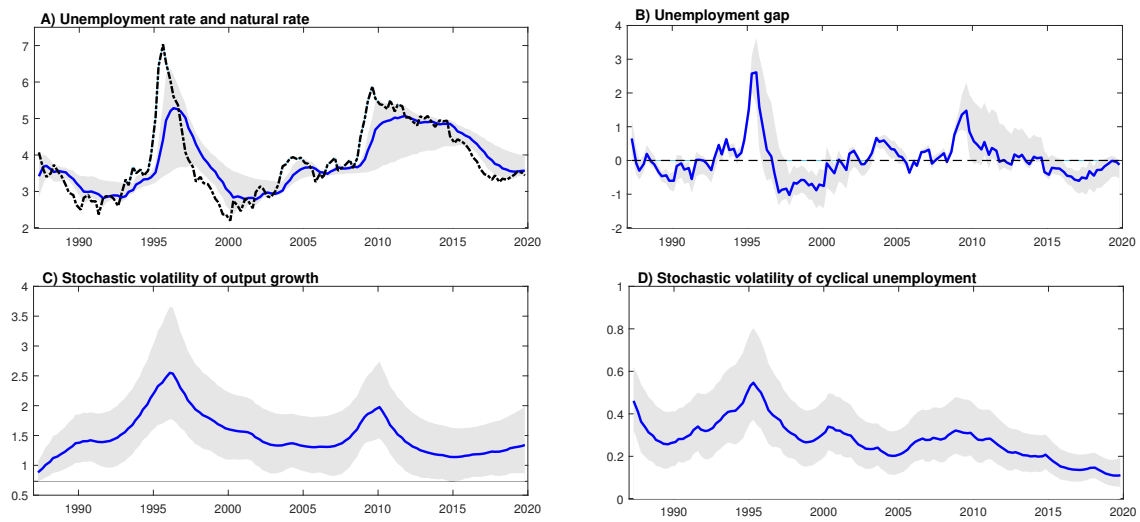

FIGURE VI: ESTIMATES OF THE NATURAL RATE, UNEMPLOYMENT GAP, AND STOCHASTIC VOLATILITIES FOR MEXICO. THE SHADED REGION REPRESENTS THE 10% AND 90% QUANTILES.

Panels A) and B) portray the unemployment rates and the natural rates superimposed on them, and the cyclical unemployment, respectively. Three main observations stand out. First, the natural rate in Mexico exhibits a high degree of variability compared to those of Brazil and Colombia. This variability is consistent with the hysteresis effects found. Second, the unemployment trajectories in these countries are varied. In Mexico, the unemployment rate shows a low level compared with the general situation in Latin America. Unemployment reaches a minimum rate of 2.2% in 2000Q1 that even outperforms the unemployment rate in the US. Loría and Ramos (2007) maintain that the official unemployment rate in Mexico can be misleading. Colombia, for its part, features a high and persistent unemployment rate of two digits during most of the sample. Consequently, the natural rate estimate shows a highly inefficient long-run equilibrium unemployment

rate. Brazil has moved from high to low unemployment during the sample. Third, the cyclical unemployment seems quite persistent in the three countries, particularly in Brazil and Colombia. Again this observation is in line with the sum of the AR lags reported above.

Panels C) and D) show the standard deviation of the innovation to the output growth and the cyclical unemployment, respectively. The two series of volatility estimates in all cases are time-varying, but they show different patterns, reflecting these countries' vicissitudes. For example, in Mexico the size of the shocks to the output growth increases substantially during the Tequila crisis and the Great financial recession. However, Colombia's output growth seems largely unaffected by the Great financial recession. Colombia's economy was severely affected by its financial crisis at the end of the 1990s.

Contrary to Colombia and Mexico, where output growth volatility estimates seem episodic, showing drastic changes during crisis episodes, in Brazil the size of the shocks has been steadily rising. A common trait among these countries is that the standard deviation of the innovation of cyclical unemployment shows a decline during the sample. There has been a slight upward drift in the size of shocks with the Tequila crisis and the Great financial recession in Mexico, the financial crisis in Colombia, and the monetary policy tightening in the late 1990s in Brazil. However, in general, the sizes of shocks have declined substantially, reaching their minimums in recent times.

5 Alternative model specification

We find that the demand variable used to identify the cyclical movements in unemployment helps us identify hysteresis only in Mexico. Concerning the latter, it is likely that hysteresis effects have not arisen in Brazil due to the small sample size. For this reason, we collect annual data on real GDP from IBGE and then we use the Chow-Lin (Chow and Lin, 1971) interpolation technique to convert the annual series of real GDP into a quarterly series dataset, using an intercept as high frequency indicator[2]. The procedure allows us to work with a larger quarterly sample size (1981Q1-2015Q4) to capture the hysteresis effects better. The larger sample size, however, is found to be irrelevant and does not change the main results since the time variability of the components is the same, Okun coefficient 95% credible interval covers positive and negative values, and there are no hysteresis effects. These results are reported in Figure XIII and Table V in appendix II.

The unobserved components model introduced in section 2 is identified using Okun's law. Another equation might have been used to specify the empirical model. For instance, the Phillips curve. We use Okun's law in an attempt to examine whether hysteresis might emerge in a context in which cyclical unemployment interacts with aggregate demand (measured by g_t). In this section, we replace equation (2.5) with the following backward-looking Phillips curve to assess the consistency of our previous results

$$\pi_t = \gamma_1 \pi_{t-1} + \gamma_2 u_t^c + \exp\{h_t^\pi\}\varepsilon_t^\pi, \quad \varepsilon_t^\pi \overset{i.i.d.}{\sim} \mathcal{N}(0,1). \tag{5.1}$$

2. Chow-Lin interpolation is a regression-based technique to transform low-frequency data into higher-frequency data. In particular, we apply the average version, which disaggregates the annual data into the means of four quarters, and select the maximum likelihood method. We use the Matlab toolbox of Quilis (2004)

This alternative exercise identifies the cyclical movements in unemployment using the Phillips curve, which might be interpreted as an aggregate supply function. In (5.1), the inflation rate changes either because of a random "cost push" term or because unemployment diverges from its natural rate (Primiceri, 2006).

The prior value for γ_2 introduced in Table I needs to be modified as this coefficient now represents the Phillips curve slope. We choose the belief of -0.5, which is a standard adoption in the literature while keeping constant the strength of that belief to 0.125. Thus, the coefficient now ranges between (-0.745,-0.255). The stochastic model specification search is employed as specified previously.

Figures X-XII show the histograms of the posterior draws of $\sigma_{\eta,\alpha}$, σ_{η,h^c}, and σ_{η,h^π} for Brazil, Colombia, and Mexico, respectively (see appendix II). The results confirm the time variability of the innovations to cyclical unemployment and inflation and the invariability of the hysteresis parameter in these countries. Table VI in appendix II reports the model parameter estimates. These estimates ratify the absence of hysteresis in Colombia, but surprisingly the results reported in Table IV for Brazil and Mexico are reverted. This alternative specification suggests that hysteresis effects are now present in Brazil and absent in Mexico. For the case of Brazil, a decrease in the cyclical unemployment rate of 1 percent leads to a permanent decrease of the actual unemployment rate of about 0.10 percent. Figures VII-IX show the states estimates under this alternative specification.

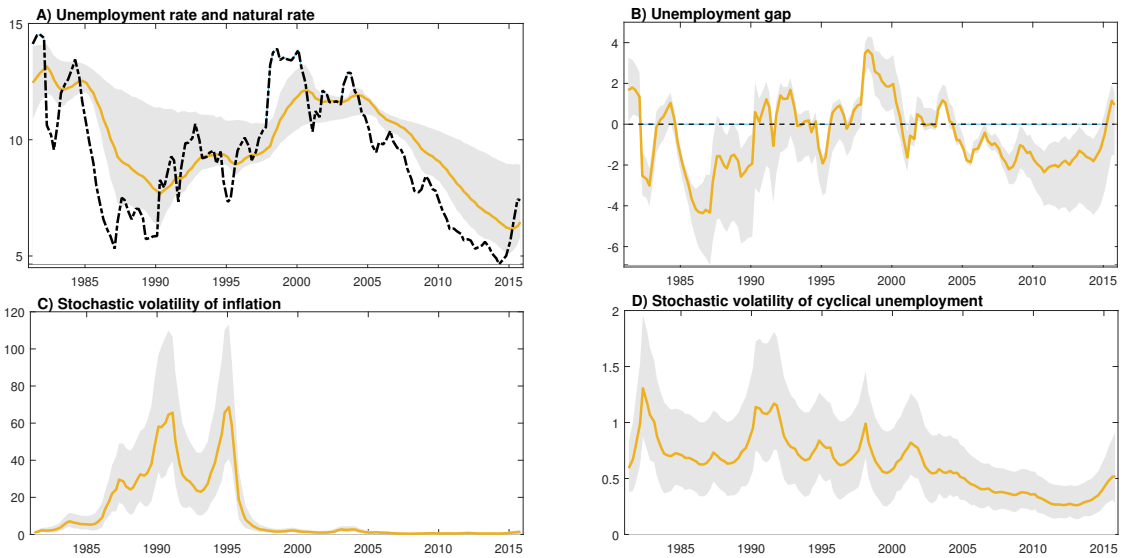

Figure VII: Estimates of the natural rate, unemployment gap, and stochastic volatilities for for Brazil. The shaded region represents the 10% and 90% quantiles.

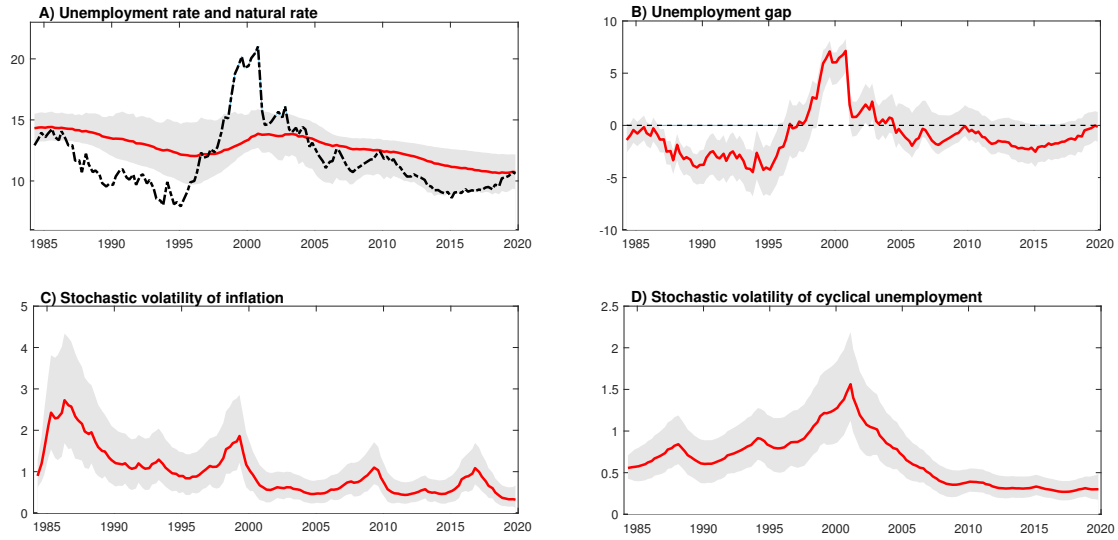

FIGURE VIII: ESTIMATES OF THE NATURAL RATE, UNEMPLOYMENT GAP, AND STOCHASTIC VOLATILITIES FOR COLOMBIA. THE SHADED REGION REPRESENTS THE 10% AND 90% QUANTILES.

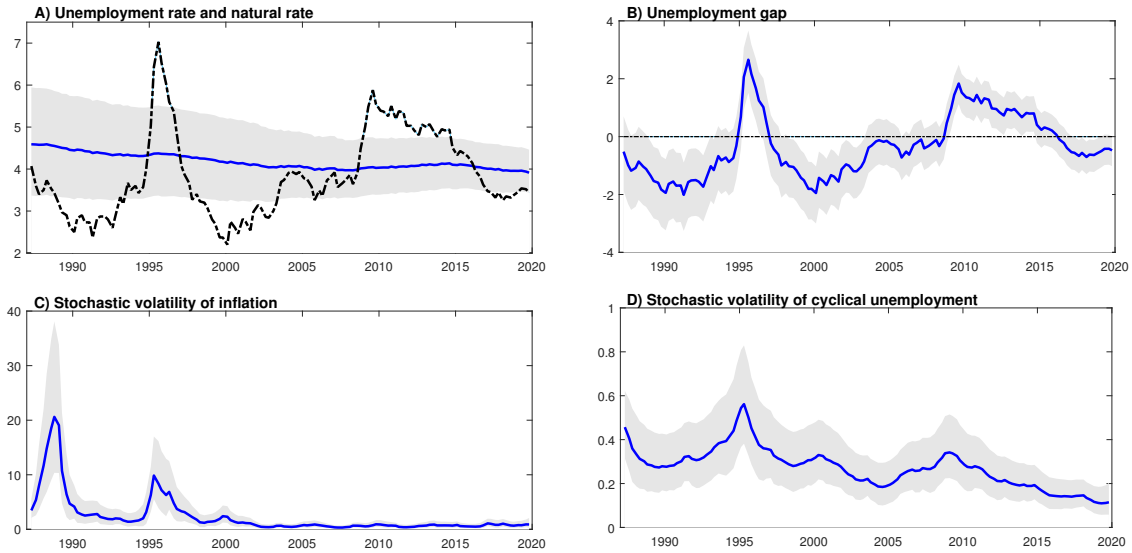

FIGURE IX: ESTIMATES OF THE NATURAL RATE, UNEMPLOYMENT GAP, AND STOCHASTIC VOLATILITIES FOR MEXICO. THE SHADED REGION REPRESENTS THE 10% AND 90% QUANTILES.

The estimates of Colombia are vividly consistent across the different specifications. On the other hand, Brazil and Mexico are the countries with the more substantive changes in terms of their unemployment dynamics. The natural rate component follows quite closely the actual unemployment rate in Brazil, as expected under hysteresis effects. In Mexico, on the contrary, the natural rate component is highly rigid with an apparent declining trend. The question now is how to reconcile the contradictory results under these two different model specifications.

One option is to assess the demand and supply variables used to identify the cyclical movements in unemployment by analyzing the coefficient that connects them to cyclical

unemployment. In other words, to assess γ_2 in (2.5) and (5.1). For the case of Brazil, the slope of the Phillips curve is -0.20, and the 95% credible interval does not contain the value of zero, while the Okun coefficient is -0.17, but the 95% credible interval includes zero. Thus, we can conclude that the inflation–cyclical employment relation is somewhat stronger than the output growth-cyclical employment relation. As for Mexico, The Okun coefficient is -1.77, and we find that the 95% credible interval excludes zero. For its part, the Phillips curve slope is -0.08, and the credible interval includes zero. Hence, we can conclude that the output growth-cyclical unemployment relation is stronger than the inflation–cyclical employment relation.

The assessment of the demand and supply variables leads us to the conclusion that cyclical movements in unemployment in Brazil and Mexico are better identified when using supply and demand specifications, respectively. Hence, hysteresis effects are also better captured in these countries when the appropriate specification is chosen. Thus, the stochastic model specification search exercise found that hysteresis effects are demand-driven in Mexico and supply-driven in Brazil, while structural determinants drive unemployment in Colombia.

5.1 Considerations on policy implications

We now turn to discuss policy implications of findings with respect to the backwardness of the productive apparatus in these countries. Colombia, the most backward country of the sample, features an unemployment rate that reflects mainly structural factors and labor market frictions. The studies on Colombian unemployment typically point to labor market friction such as the high and inflexible minimum wage and the high non-wage labor costs that create disemployment effects and raise the costs to hire new workers, as the prime suspects for the high unemployment (see, e.g., Kugler (2004), and Santamaría et al. (2010)). Those studies advocate for a higher labor market flexibility, and the typical policy recommendation is to reduce the minimum wage and eliminate the non-wage labor costs.

The policy recommendation of these studies tends to emphasize frictions rather than structural factors associated with the relative backwardness of Colombia. A recent work of García and Cruz (2017) shows that reforms designed to make the labor market more flexible in Latin America have resulted insignificant to reduce unemployment. Their work indicates that capital accumulation is the variable that explains the majority of the behavior of unemployment in the region. In the same vein, Ros (2005) shows that those Latin American countries that have performed better in unemployment are those that increased their ratios of manufacturing exports and manufacturing employment. He also finds that the increase in total investment that stimulates the accumulation of capital stocks generates a double beneficial effect on employment because it increases the productivity and production of the formal sectors.

García and Cruz (2017) and Ros (2005) findings connect directly with the development economics, as it appeared in the 1940s and 1950s. This theory remains heuristically necessary for its intuition that serves as a basis for theoretical work relevant to understanding structural unemployment. For example, Lewis's (1954) model is perhaps the one most relevant in a developing-country context. Under the Lewisian perspective of a dual economy, the high minimum wage with respect to the median income is not a cause; it is a symptom of high unemployment and informality.[3] The cause of a high unemploy-

3. The OECD (2019) reports that in Colombia, the national minimum wage is 86% of the median

ment rate and informality is a low labor productivity, which is ultimately determined by capital formation. A higher capital accumulation increases labor productivity, raises median income, and draws labor from low productivity sectors toward high productivity activities, reducing thus unemployment and informality. Thus, policy recommendations in Colombia should aim to facilitate capital accumulation to heighten productivity and incomes, and reduce unemployment and informality.

Brazilian unemployment hysteresis seems to be dominantly driven by the supply-side of the economy. In developing economies like Brazil, where labor is an irrelevant constraint due to large amounts of hidden unemployment and underemployment, capital supply becomes the most important constraint (Skott, 2021). Hence, stabilization policies should aim to smooth the pace of capital accumulation. Skott (2021) suggests that adaptations of "functional finance" to developing-country context might help stabilize the level and composition of demand at values that are consistent with a target rate of capital accumulation of the modern sectors. Finally, in the relatively advanced Mexico, where unemployment is low, and demand forces account for a substantial part of unemployment dynamics, macroeconomic stabilization policies such as countercyclical monetary and fiscal policies and prudential policies to mitigate crises gain importance.

6 Conclusion

In this document, we inquired about the nature of the unemployment rate in three major Latin American economies, namely Brazil, Colombia, and Mexico. With the help of a Bayesian model selection approach, we distinguish between the hypotheses of hysteresis and structural unemployment. The model selection exercise suggests that the unemployment rate in Colombia is merely structural, while we found substantial hysteresis effects in Brazil and Mexico. Unlike what has been reported before for developed economies, time-varying hysteresis is not crucial in the cases of these three countries. The most important finding that emerges from this study is that hysteresis effects found are varied in nature. In Mexico, demand drivers propelled hysteresis effects, and in Brazil, those effects are fostered by supply determinants.

The Bayesian model selection exercise also found that stochastic volatility is vital for studying cyclical unemployment, output growth, and inflation in these countries. The volatility of the innovations to these components shows a substantial time variability, and they seem to capture well the rise of the size of the shocks during crisis episodes.

Since the unemployment rate in Colombia has structural roots, we consider the classical literature of development economics. This branch of literature provides important lessons to understand why developing countries can exhibit protracted periods of high unemployment. Based on the Lewisian model, we suggest that capital accumulation might be essential to reduce the Colombian unemployment rate in the long run. In Brazil, fiscal policy might be used to align aggregate demand to the requirements of capital accumulation in moderns sectors. Finally, for Mexico, we suggest that macro-prudential policies might be employed to stabilize the aggregate demand.

wage. This proportion indicates that the minimum wage is an effective wage in Colombia, as previously shown by Bell (1997).

Appendix I

This appendix discusses the Gibbs MCMC sampler, which rests on the Berger et al. (2016) implementation of the Frühwirth-Schnatter and Wagner (2010) algorithm. As in Berger et al. (2016), we adopt an equation-by-equation basis to draw from the posterior of interest introduced in subsection 3.3. More specifically, consider the general regression model

$$y = zb + e, \quad e \sim \mathcal{N}(0, \Sigma), \tag{I.1}$$

with y a vector including observations on a dependent variable of interest, z an unrestricted predictor matrix with rows that contain the state processes that are relevant for explaining y, and b as the corresponding unrestricted parameter vector is denoted . Furthermore, Σ is a diagonal matrix that may vary over time to allow for heteroscedasticity.

Heteroscedasticity is modeled in the form of stochastic volatility, $\exp\{h_t^i\}$, which is non-linear but can be easily linearized by taking the logarithm of its square

$$\ln\left(\exp\left\{h_t^i\right\}\varepsilon_t^i\right)^2 = 2h_t^i + \ln\left(\varepsilon_t^i\right)^2,$$

where $\ln\left(\varepsilon_t^i\right)^2$ is log-chi-square distributed with expected value -1.2704 and variance $3.1416^2/2$. Following Kim et al. (1998), we approximate the linear model by an offset mixture time series model as

$$l_t^i = 2h_t^i + \xi_t^i,$$

where $l_t^i = \ln\left(\left(\exp\{h_t^i\}\varepsilon_t^i\right)^2 + d\right)$, for $i = c, g(\pi)$ with $d = 0.001$ being a number added to induce stability by ensuring the term inside the ln is bounded away from zero. The functions l_t^i are computed as

$$l_t^c = \ln\left((u_t^c - \beta_1 u_{t-1}^c - \beta_2 u_{t-2}^c)^2 + 0.001\right),$$

and

$$l_t^g = \ln\left((g_t - \gamma_1 g_{t-1} - \gamma_2 u_t^c)^2 + 0.001\right).$$

Next, we approximate the distribution of ξ_t^i by the following mixture of normals

$$f\left(\xi_t^i\right) = \sum_{k=1}^{M} q_k f_N\left(\xi_t^i \middle| \, m_k - 1.2704, \nu_k^2\right),$$

with component probabilities q_k, means $m_k - 1.2704$, and variances ν_k^2. Following Omori et al. (2007), we use a mixture of $M = 10$ normal distributions to make the approximation to the log-chi-square distribution. Values for $\{q_k, m_k, \nu_k^2\}$ are provided by Omori et al. (2007) in their Table 1. The resulting model is now linear and Gaussian, which permits the use of the Kalman filter and smoother.

The unobserved components model introduced in section 2 admits a state-space representation. Our Gibbs MCMC sampler exploits this state space representation to create posterior draws of the time-varying parameter α_t and ψ_t using Kalman filtering and Kalman smoothing based on the multimove simulation smoother of Carter and Kohn (1994) and De Jong and Shephard (1995). The state-space representation is

$$
\begin{aligned}
y_t &= z_t S_t + e_t, \quad e_t \overset{i.i.d.}{\sim} \mathcal{N}(0, H_t), \\
S_{t+1} &= R S_t + K_t v_t, \quad v_t \overset{i.i.d.}{\sim} \mathcal{N}(0, Q_t), \quad S_1 \overset{i.i.d.}{\sim} \mathcal{N}(a_1, A_1)
\end{aligned}
$$

with y_t a vector of observations and S_t an unobserved state vector. The matrices z_t, R, K_t, H_t, and Q_t are known. The initial state vector, S_1, is initialized with mean a_1 and variance A_1.

The expanded version of the state space representation of our empirical model is given by

$$
\text{(I.2)} \qquad \overbrace{\begin{bmatrix} u_t \\ g_t - \gamma_1 g_{t-1} \end{bmatrix}}^{y_t} = \overbrace{\begin{bmatrix} 1 & 1 & 0 \\ 0 & \gamma_2 & 0 \end{bmatrix}}^{z_t} \overbrace{\begin{bmatrix} u_t^\tau \\ u_t^c \\ u_{t-1}^c \end{bmatrix}}^{S_t} + \overbrace{\begin{bmatrix} 0 \\ \exp\{h_t^g\} \varepsilon_t^g \end{bmatrix}}^{e_t}
$$

$$
\text{(I.3)} \qquad \underbrace{\begin{bmatrix} u_{t+1}^\tau \\ u_{t+1}^c \\ u_t^c \end{bmatrix}}_{S_{t+1}} = \underbrace{\begin{bmatrix} 1 & \alpha_t & 0 \\ 0 & \beta_1 & \beta_2 \\ 0 & 1 & 0 \end{bmatrix}}_{R} \underbrace{\begin{bmatrix} u_t^\tau \\ u_t^c \\ u_{t-1}^c \end{bmatrix}}_{S_t} + \underbrace{\begin{bmatrix} \sigma_{\eta,\tau} & 0 \\ 0 & \exp\{h_t^c\} \\ 0 & 0 \end{bmatrix}}_{K_t} \underbrace{\begin{bmatrix} \eta_t^\tau \\ \eta_t^c \end{bmatrix}}_{v_t}
$$

with $H_t = diag(0, \exp\{h_t^g\})$, $a_1 = 0$, $A_1 = 1000$, and $Q_t = I_2$.

Likewise, the state space representation for the time-varying hysteresis parameter is given by

$$
\text{(I.4)} \qquad \overbrace{\left[u_{t+1}^\tau - u_t^\tau - \alpha_0 u_t^c\right]}^{y_t} = \overbrace{\left[\sigma_{\eta,\alpha} u_t^c\right]}^{z_t} \overbrace{\left[\widetilde{\alpha}_t\right]}^{S_t} + \overbrace{\left[\eta_t^\tau\right]}^{e_t}
$$

$$
\text{(I.5)} \qquad \underbrace{\left[\widetilde{\alpha}_{t+1}\right]}_{S_{t+1}} = \underbrace{\left[1\right]}_{R} \underbrace{\left[\widetilde{\alpha}_t\right]}_{S_t} + \underbrace{\left[1\right]}_{K_t} \underbrace{\left[\widetilde{\epsilon}_t\right]}_{v_t}
$$

with $H_t = \sigma_{\eta,\tau}^2$ and $Q = 1$.

The state-space representation of the non-centered parameterization of the stochastic volatility is given by:

$$
\text{(I.6)} \qquad \overbrace{\left[l_t^i - (m_k - 1.2704) - 2h_0^i\right]}^{y_t} = \overbrace{\left[2\theta^i \sigma_{\eta,h^i}\right]}^{z_t} \overbrace{\left[\widetilde{h}_t^{\,i}\right]}^{S_t} + \overbrace{\left[\widetilde{\xi}_t^{\,i}\right]}^{e_t}
$$

$$
\text{(I.7)} \qquad \underbrace{\left[\widetilde{h}_{t+1}^i\right]}_{S_{t+1}} = \underbrace{\left[1\right]}_{R} \underbrace{\left[\widetilde{h}_t^i\right]}_{S_t} + \underbrace{\left[1\right]}_{K_t} \underbrace{\left[\widetilde{\epsilon}_t^i\right]}_{v_t}
$$

The following algorithm has details about our Gibbs MCMC sampler:

1. Run the Kalman filter to generate $\{\psi_{t|t}\}_{t=1}^T$, given the state-space representation (I.2)-(I.3).

2. Draw $\breve{\psi}_{T|T}$, which is input into the Kalman smoother to generate $\breve{\psi}_{T-1|T}$. Continue iterating the Kalman smoother backwards in time to create the smoothed candidate

draw $\breve{\psi}^T = \{\breve{\psi}_{t|T}\}_{t=1}^T$.

3. Run the Kalman filter to generate $\{\tilde{\alpha}_{t|t}\}_{t=1}^T$, given the state-space representation (I.4)-(I.5).

4. Draw $\breve{\tilde{\alpha}}_{T|T}$, which is input into the Kalman smoother to generate $\breve{\tilde{\alpha}}_{T-1|T}$. Continue iterating the Kalman smoother backwards in time to create the smoothed candidate draw $\breve{\tilde{\alpha}}^T = \{\breve{\tilde{\alpha}}_{t|T}\}_{t=1}^T$. Note that in the case of a time invariant hysteresis parameter no forward-filtering and backward-sampling is needed.

5. Since ε_t^i, for $i = c, g(\pi)$ is Gaussian, draw the stochastic volatilities from the 10-component mixture of Omori et al. (2007) introduced above and the state-space representation (I.6)-(I.7).

6. Sample β_1, β_2, γ_1, and γ_2 from equation (I.1). In the case of β_1 and β_2, $y = u_t^c$, $z = (u_{t-1}^c, u_{t-2}^c)$, $b = (\beta_1, \beta_2)'$ and $e = \exp\{h_t^c\}\eta_t^c$, where $\Sigma = \text{diag}(\exp\{h_1^c\}, ..., \exp\{h_T^c\})$. As for γ_1 and γ_2, $y = g_t$, $z = (g_{t-1}, u_t^c)$, $b = (\gamma_1, \gamma_2)'$ and $e = \exp\{h_t^g\}\eta_t^g$, where $\Sigma = \text{diag}(\exp\{h_1^g\}, ..., \exp\{h_T^g\})$.

7. Sample the binary indicator θ^i, and the parameters h_0^i and σ_{η,h^i} for $i = c, g(\pi)$, using the general linear regression format of (I.1) such that $y = l_t^i - (m_k - 1.2704)$, $z = 2(1, \theta^i \tilde{h}_t^i)$, $b = (h_0^i, \sigma_{\eta,h^i})$, and $e = \tilde{\xi}_t^i$ with $\tilde{\xi}_t^i = \xi_t^i - (m_k - 1.2704)$. The binary indicator θ^i is sampled from the Bernoulli distribution with probability $p(\theta^i) = \frac{f(\theta^i=1)}{f(\theta^i=0)+f(\theta^i=1)}$.

8. Sample the binary indicator λ, and the parameters α_0, and $\sigma_{\eta,\alpha}$, using the general linear regression format of (I.1) with $y = u_t^\tau - u_{t-1}^\tau$, $z = (u_{t-1}^c, \lambda\tilde{\alpha}_t u_{t-1}^c)$, $b = (\alpha_0, \sigma_{\eta,\alpha})'$ and $e = \eta_t^\tau$, where $\Sigma = \sigma_e^2 I_T$. The binary indicator λ is sampled from the Bernoulli distribution as above.

9. The conditional distribution of $\sigma_{\eta,\tau}^2$ is non-standard due to the gamma prior. However, it can be sampled using a Metropolis-Hastings step with an inverted gamma ($\mathcal{IG}$) proposal density. Following Grant and Chan (2017), we first obtain a candidate draw s^2 from $\mathcal{IG} = (T/2-1, \sum_{t=2}^T (u_{t+1}^\tau - u_t^\tau - \alpha_t u_t^c)^2/2)$. Given the current draw $\sigma_{\eta,\tau}^2$, we accept the candidate draw s^2 with probability $\min\{1, \exp(-\frac{1}{2V_{\sigma_{\eta,\tau}}}(s^2 - \sigma_{\eta,\tau}^2))\}$; otherwise, we keep $\sigma_{\eta,\tau}^2$.

10. Randomly permute the signs of $\sigma_{\eta,\alpha}$ and $\widetilde{\alpha}_t$, and of σ_{η,h^i} and $\widetilde{h}_t^i$ for $i = c,\ g(\pi)$.

11. Repeat steps 1 to 10 to obtain $\mathcal{J}$ draws

Appendix II

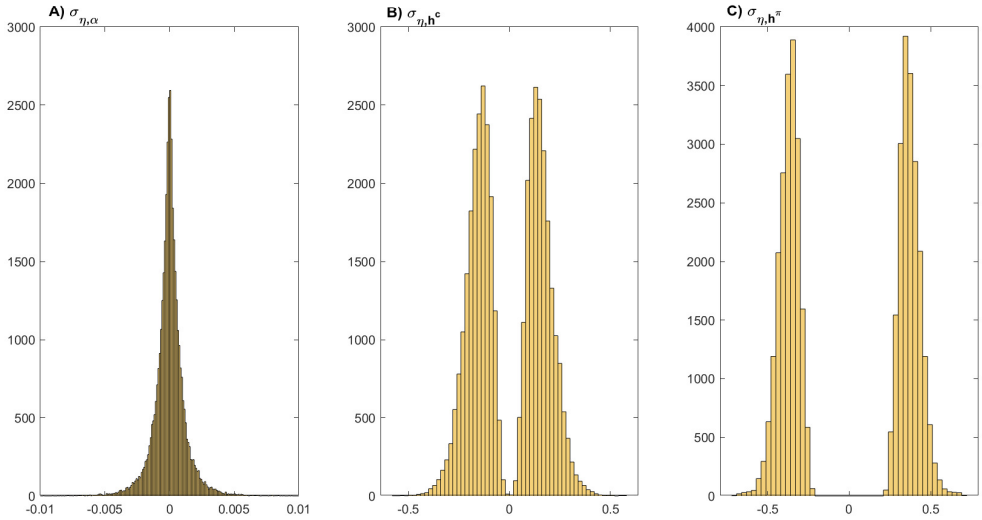

Figure X: Histograms of the posterior draws for the standard deviations of the innovations to the hysteresis parameter and stochastic volatilities for Brazil

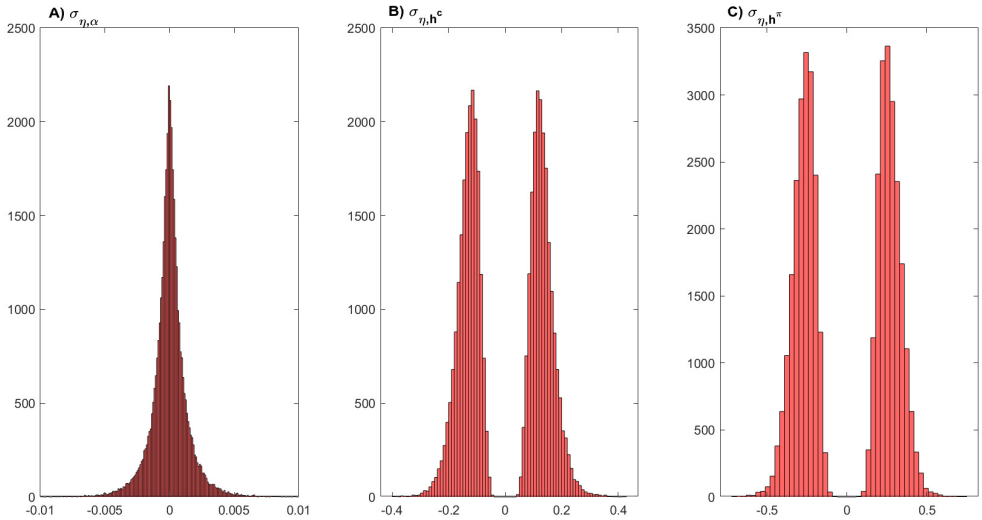

Figure XI: Histograms of the posterior draws for the standard deviations of the innovations to the hysteresis parameter and stochastic volatilities for Colombia

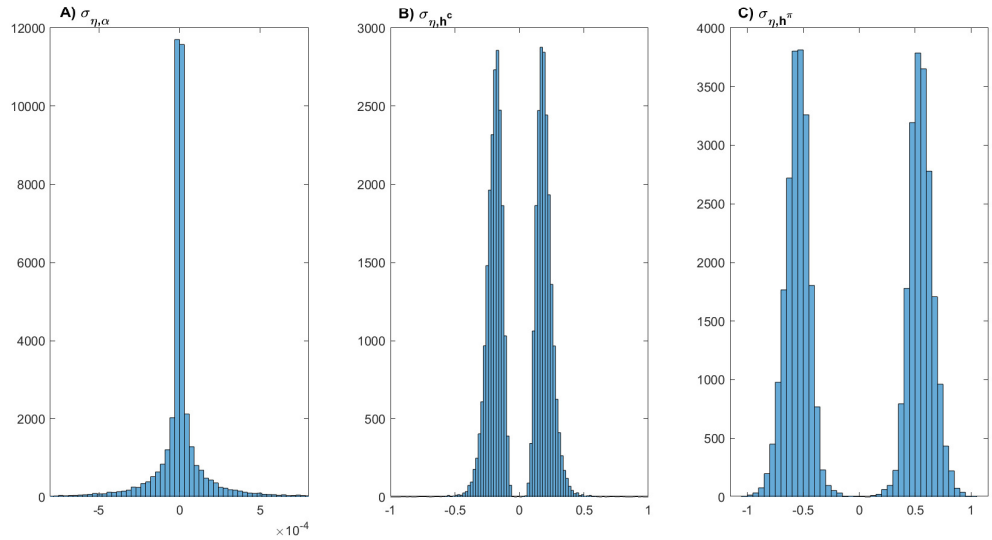

Figure XII: Histograms of the posterior draws for the standard deviations of the innovations to the hysteresis parameter and stochastic volatilities for Mexico

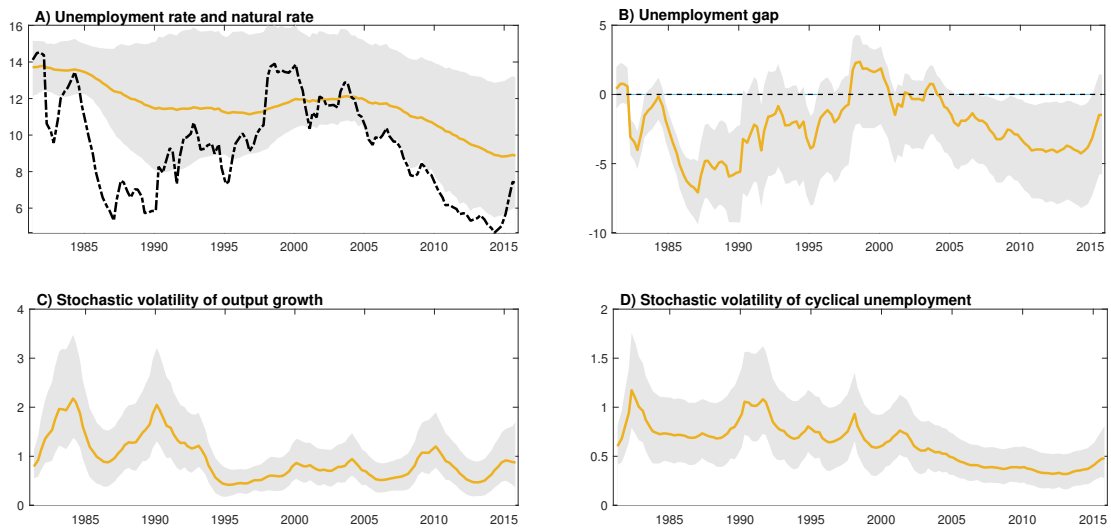

Figure XIII: Estimates of the natural rate, unemployment gap, and stochastic volatilities for Brazil using a bigger sample. The shaded region represents the 10% and 90% quantiles.

References

Ayala, Astrid, Juncal Cuñado, and Luis Albériko Gil-Alana (2012), "Unemployment hysteresis: Empirical evidence for latin america." *Journal of Applied Economics*, 15, 213–233, URL https://doi.org/10.1016/S1514-0326(12)60010-5.

TABLE V: Posterior distributions of model parameters using a bigger sample for Brazil

		Brazil		
		Mean	Percentiles	
Parameters:			2.5%	97.5%
1st AR lag of unemployment gap	β_1	1.26	1.18	1.33
2nd AR lag of unemployment gap	β_2	-0.32	-0.39	-0.24
AR(1) lag of output growth	γ_1	0.85	0.78	0.91
Okun coefficient	γ_2	-0.04	-0.16	0.02
Constant hysteresis coefficient	α_0	0.01	-0.03	0.06
Constant volatility of cyclical unemployment	$\exp\{h_0^c\}$	0.55	0.45	0.66
Constant volatility of output growth	$\exp\{h_0^g\}$	0.61	0.51	0.73
std. of time-varying hysteresis coefficient	$\sigma_{\eta,\alpha}$	0.00	0.00	0.00
std. of SV: cyclical unemployment	σ_{η,h^c}	0.002	-0.28	0.28
std. of SV: output growth	σ_{η,h^g}	0.001	-0.35	0.35
std. of the natural rate	$\sigma_{\eta,\tau}$	0.52	0.37	0.66

TABLE VI: Posterior distributions of model parameters

		Brazil			**Colombia**			**Mexico**		
		Mean	Percentiles		Mean	Percentiles		Mean	Percentiles	
Parameters:			2.5%	97.5%		2.5%	97.5%		2.5%	97.5%
1st AR lag of unemployment gap	β_1	1.24	1.16	1.31	1.16	1.09	1.24	1.22	1.15	1.30
2nd AR lag of unemployment gap	$beta_2$	-0.32	-0.40	-0.25	-0.23	-0.31	-0.16	-0.30	-0.38	-0.23
AR(1) lag of inflation	γ_1	1.00	0.99	1.005	0.98	0.96	1.003	0.99	0.98	1.00
Phillips curve slope	γ_2	-0.20	-0.39	-0.04	-0.05	-0.11	0.005	-0.08	-0.20	0.02
Constant hysteresis coefficient	α_0	0.10	0.008	0.21	0.01	-0.03	0.06	0.00	0.00	0.00
Constant volatility of cyclical unemployment	$\exp\{h_0^c\}$	0.54	0.44	0.64	0.55	0.46	0.64	0.53	0.44	0.63
Constant volatility of inflation	$\exp\{h_0^\pi\}$	0.57	0.44	0.69	0.59	0.49	0.71	0.65	0.50	0.80
std. of time-varying hysteresis coefficient	σ_{η,h^α}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
std. of SV: cyclical unemployment	σ_{η,h^c}	0.001	-0.33	0.33	0.001	-0.19	0.19	0.0004	-0.27	0.27
std. of SV: inflation	σ_{η,h^g}	0.002	-0.47	0.47	0.002	-0.36	0.36	0.001	-0.68	0.68
std. of the natural rate	σ_{η,h^τ}	0.60	0.49	0.73	0.21	0.10	0.32	0.002	0.0008	0.008

Ball, Laurence, Nicolás De Roux, and Marc Hofstetter (2013), "Unemployment in latin america and the caribbean." *Open Economies Review*, 24, 397–424, URL https://doi.org/10.1007/s11079-012-9248-2.

Ball, Laurence M (2009), "Hysteresis in unemployment: Old and new evidence." Working Paper 14818, National Bureau of Economic Research, URL http://www.nber.org/papers/w14818.

Bell, Linda A. (1997), "The impact of minimum wages in mexico and colombia." *Journal of Labor Economics*, 15, S102–S135, URL http://www.jstor.org/stable/10.1086/209878.

Berger, Tino and Gerdie Everaert (2008), "Unemployment persistence and the nairu: A bayesian approach." *Scottish Journal of Political Economy*, 55, 281–299, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1467-9485.2008.00454.x.

Berger, Tino, Gerdie Everaert, and Hauke Vierke (2016), "Testing for time variation in an unobserved components model for the u.s. economy." *Journal of Economic Dynamics and Control*, 69, 179–208, URL https://www.sciencedirect.com/science/article/pii/S016518891630094X.

Berger, Tino and Bernd Kempa (2019), "Testing for time variation in the natural rate of interest." *Journal of Applied Econometrics*, 34, 836–842, URL https://onlinelibrary.wiley.com/doi/abs/10.1002/jae.2698.

Blanchard, Olivier J and Lawrence H Summers (1986), "Hysteresis and the european unemployment problem." Working Paper 1950, National Bureau of Economic Research, URL http://www.nber.org/papers/w1950.

Carter, C. K. and R. Kohn (1994), "On gibbs sampling for state space models." *Biometrika*, 81, 541–553, URL www.scopus.com. Cited By :884.

Chan, Joshua C.C. and Rodney W. Strachan (2021), "Bayesian state space models in macroeconometrics." *Journal of Economic Surveys*, n/a, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/joes.12405.

Chow, Gregory C. and An-Loh Lin (1971), "Best linear unbiased interpolation, distribution, and extrapolation of time series by related series." *The Review of Economics and Statistics*, 53, 372–375, URL http://www.jstor.org/stable/1928739.

Cogley, Timothy, Giorgio E. Primiceri, and Thomas J. Sargent (2010), "Inflation-gap persistence in the us." *American Economic Journal: Macroeconomics*, 2, 43–69, URL https://www.aeaweb.org/articles?id=10.1257/mac.2.1.43.

Cogley, Timothy and Thomas J. Sargent (2005), "Drifts and volatilities: monetary policies and outcomes in the post wwii us." *Review of Economic Dynamics*, 8, 262–302, URL https://www.sciencedirect.com/science/article/pii/S1094202505000049. Monetary Policy and Learning.

Correa, Victor, Antonio Escandón, René Luengo, and José Venegas (2003), "Empalme de Series Anuales y Trimestrales del PIB." *Notas de Investigación Journal Economía Chilena (The Chilean Economy)*, 6, 77–86, URL https://ideas.repec.org/a/chb/bcchni/v6y2003i1p77-86.html.

De Jong, P. and N. Shephard (1995), "The simulation smoother for time series models." *Biometrika*, 82, 339–350, URL www.scopus.com. Cited By :268.

Frühwirth-Schnatter, Sylvia and Helga Wagner (2010), "Stochastic model specification search for gaussian and partial non-gaussian state space models." *Journal of Econometrics*, 154, 85–100, URL https://www.sciencedirect.com/science/article/pii/S0304407609001614.

Galí, Jordi and Luca Gambetti (2009), "On the sources of the great moderation." *American Economic Journal: Macroeconomics*, 1, 26–57, URL https://www.aeaweb.org/articles?id=10.1257/mac.1.1.26.

García, Luis Fernando and Moritz Cruz (2017), "Desempleo en américa latina: ¿flexibilidad laboral o acumulación de capital?" *Problemas del Desarrollo*, 48, 33–56, URL https://www.sciencedirect.com/science/article/pii/S0301703617300159.

Grant, Angelia and Joshua Chan (2017), "Measuring the output gap using stochastic model specification search." Working Paper 2/2017, Crawford School of Public Policy, URL https://cama.crawford.anu.edu.au/sites/default/files/publication/cama_crawford_anu_edu_au/2017-01/2_2017_chan_grant.pdf.

Grant, Angelia L. (2018), "The great recession and okun's law." *Economic Modelling*, 69, 291–300, URL https://www.sciencedirect.com/science/article/pii/S0264999317308799.

Jaeger, Albert and Martin Parkinson (1994), "Some evidence on hysteresis in unemployment rates." *European Economic Review*, 38, 329–342, URL https://www.sciencedirect.com/science/article/pii/0014292194900612.

Kim, S., N. Shephard, and S. Chib (1998), "Stochastic volatility: Likelihood inference and comparison with arch models." *Review of Economic Studies*, 65, 361–393, URL www.scopus.com. Cited By :990.

Kugler, Adriana D. (2004), *Law and employment lessons from Latin America and the Caribbean*, chapter The Effect of Job Security Regulations on Labor Market Flexibility: Evidence from the Colombian Labor Market Reform. University of Chicago Press.

Laubach, Thomas (2001), "Measuring the nairu: Evidence from seven economies." *The Review of Economics and Statistics*, 83, 218–231, URL http://www.jstor.org/stable/3211601.

León–Ledesma, Miguel A. (2002), "Unemployment hysteresis in the us states and the eu: A panel approach." *Bulletin of Economic Research*, 54, 95–103, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/1467-8586.00141.

Lewis, W. Arthur (1954), "Economic development with unlimited supplies of labour." *The Manchester School*, 22, 139–191, URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1467-9957.1954.tb00021.x.

Loría, Eduardo and Manuel G. Ramos (2007), "La ley de okun: una relectura para méxico, 1970-2004." *Estudios Económicos*, 22, 19–55, URL http://www.jstor.org/stable/40311523.

Nelson, Charles R. and Charles R. Plosser (1982), "Trends and random walks in macroeconmic time series: Some evidence and implications." *Journal of Monetary Economics*, 10, 139–162, URL https://www.sciencedirect.com/science/article/pii/0304393282900125.

OECD (2019), "Oecd economic surveys: Colombia 2019." Economic survey, Organisation for Economic Co-operation and Development, URL https://www.oecd.org/economy/surveys/Colombia-2019-OECD-economic-survey-overview.pdf.

Omori, Yasuhiro, Siddhartha Chib, Neil Shephard, and Jouchi Nakajima (2007), "Stochastic volatility with leverage: Fast and efficient likelihood inference." *Journal of Econometrics*, 140, 425–449, URL https://www.sciencedirect.com/science/article/pii/S0304407606001436.

Pérez-Alonso, Alicia and Silvestro Di Sanzo (2010), "Unemployment and hysteresis: A nonlinear unobserved components approach." *Studies in Nonlinear Dynamics & Econometrics*, 15, URL https://doi.org/10.2202/1558-3708.1806.

Primiceri, Giorgio E. (2005), "Time Varying Structural Vector Autoregressions and Monetary Policy." *The Review of Economic Studies*, 72, 821–852, URL https://doi.org/10.1111/j.1467-937X.2005.00353.x.

Primiceri, Giorgio E. (2006), "Why Inflation Rose and Fell: Policy-Makers' Beliefs and U. S. Postwar Stabilization Policy*." *The Quarterly Journal of Economics*, 121, 867–901, URL https://doi.org/10.1162/qjec.121.3.867.

Quilis, Enrique M. (2004), "A matlab library of temporal disaggregation and interpolation methods, instituto nacional de estadística, internal document, madrid, spain."

Rissman, Ellen (1986), "What is the natural rate of unemployment?" *Economic Perspectives*, 10, 3–17, URL https://www.chicagofed.org/publications/economic-perspectives/1986/september-october-rissman.

Ros, Jaime (2005), "El desempleo en américa latina desde 1990." Working Paper 29, Naciones Unidas, CEPAL, Unidad de Desarrollo Económico, URL http://digitallibrary.un.org/record/550923.

Santamaría, Mauricio, Roberto Steiner, and Erika Schutt (2010), *Colombia 2010-2014: propuestas de política pública*, chapter ¿Cómo derrotar el desempleo y la informalidad? Fedesarrollo, Bogotá.

Scott, James G. and James O. Berger (2010), "Bayes and empirical-Bayes multiplicity adjustment in the variable-selection problem." *The Annals of Statistics*, 38, 2587 – 2619, URL https://doi.org/10.1214/10-AOS792.

Sims, Christopher A. and Tao Zha (2006), "Were there regime switches in u.s. monetary policy?" *American Economic Review*, 96, 54–81, URL https://www.aeaweb.org/articles?id=10.1257/000282806776157678.

Skott, Peter (2021), "Fiscal policy and structural transformation in developing economies." *Structural Change and Economic Dynamics*, 56, 129–140, URL https://www.sciencedirect.com/science/article/pii/S0954349X20304070.

Thirlwall, Anthony Philip (1979), "The Balance of Payments Constraint as an Explanation of International Growth Rate Differences." *BNL Quarterly Review*, 32, 45–53, URL https://ideas.repec.org/a/psl/bnlaqr/197901.html.